Problem-based learning's impact on the understanding of place value

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The conceptual development of place value is often described as the most significant of all number concepts which, when understood, enables students to develop a comprehensive sense of number (Reys, Lindquist, Lambdin, & Smith, 2009). Disappointingly, the development of place value is seen as an elusive concept for pre-service teachers. This paper reports on a study to determine the effect of a problem-based learning (PBL) approach versus a traditional teacher-centred instructional approach in a university course on pre-service teacher's content knowledge and conceptual understanding of place value.

Keywords: problem-based learning, mathematics education, place value

Introduction

Globally, current educational reforms and initiatives prioritise literacy and numeracy outcomes for students. In 2008, the Australian Ministerial Council on Education stated, "Literacy and numeracy ... [are] the cornerstones of schooling for young Australians"(p. 5). Moreover, in support of a national set of educational goals, "The Australian mathematics curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought processes and problem-solving skills." (Australian Curriculum Assessment, 2010, p. 1). Accordingly, the responsibility for achieving these and other goals related to sustaining Australia's growth and prosperity in a competitive global economy is placed with teachers. Not surprisingly, teacher's professional knowledge bases (Shulman, 1987) have come under scrutiny in an attempt to understand the work of teachers and the global changes that have affected the way that teachers go about their work (Exley, 2005).

Miller (2003) states, "Effective teaching begins with effective teacher preparation (p. 3)." Haystead and Marzano (2009) also advise that university teacher preparation programs should focus their efforts on ensuring that graduates have strong content knowledge and are equipped to use research-based instructional strategies. Miller (2003) further suggests that by integrating researched-based teaching and learning strategies, teachers would be better equipped to bring conceptual understanding to the forefront of student learning. Miller's study of the American education system "reveals a 39 percentage-point difference in student achievement between students with 'most effective' teachers and 'least effective' teachers" (p. 2).

The Marzano Research Laboratory (Haystead M. W. & Marzano R. J., 2009) employed a meta-analysis to investigate instructional strategies over a five-year period which involved 300+ volunteer teachers, 38 schools in 14 school districts in the United States. Their principle findings revealed that, on average, when particular instructional strategies are used, there is a 16% learning gain between the students' pre-test and the post-test. Teaching strategies having

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such a statistically significant impact on student learning certainly warrants identification. Three of the instructional strategies investigated that are of particular interest to this paper align with the principles of problem-based learning.

Problem-based learning (PBL)

The literature on PBL originated from the medical field in the mid-1950's. PBL has since been adopted by many other professions including schools of education (Savery & Duffy, 1996). Of particular interest to education is Major & Palmer's (2001, p. 1) definition:

> *PBL is an educational approach in which complex problems serve as the context and the stimulus for learning. In PBL classes, students work in teams to solve one or more complex and compelling "real world" problems. They develop skills in collecting, evaluating, and synthesizing resources as they first define and then propose a solution to a multi-faceted problem.*

Levin (2001) also suggests that PBL is an instructional method which promotes critical thinking and problem-solving skills within the context of real-world problems which result in the acquisition of the intended content and conceptual knowledge. Furthermore, PBL instruction is considerably more student-centred than teacher directed.

Terms such as experiential, discovery, student-centred and inquiry learning (IL) have become synonymous with the PBL approach. Some have categorised these terms together as examples of the constructivist approach and placed PBL under the umbrella of minimally guided instruction (Kirschner, Sweller, & Clark, 2006). However, on the contrary, PBL requires skilled scaffolding of the students' learning by the teacher. To imply that PBL should be placed under the umbrella of minimally guided instruction does not accurately describe the instructional process. More accurately, PBL should be considered consistent with the principles of effective instruction *arising from* constructivism.

Hemelo-Silver, Duncan, and Chinn (2007) also disagree with the notion of classifying these pedagogical approaches under the category of minimally guided instruction and argue that these approaches, in particular, PBL and IL, are not minimally guided instructional approaches but rather offer extensive scaffolding and direction to support student learning (p. 99). For example, under the PBL model, the teacher facilitates the learning process by challenging the students' thinking through the asking of key, high order questions which probe deeply into what students know. This information together with additional investigations would be analysed and then synthesised by the students into new coherent forms of understanding required to solve the set problems. The idea being that their learning becomes connected rather than isolated and cumulative rather than fragmented (Levin, 2001).

Over the duration of a PBL session, the teacher wears many hats and plays several roles such as lecturer, facilitator, coach, evaluator, assessor and supporter. The facilitator has a responsibility to avoid using his or her own content knowledge when asking questions which only serve to evoke a 'correct answer'. The objective of this avoidance is to move the students toward self-discovery of the desired outcome thus allowing them to *own the knowledge* versus being fed the information and/or the solution through the traditional teacher-led instructional approach. In this way, as a real-world or prearranged contextual problem whose solution is not immediately apparent is presented, the students become the owner of the work and assume responsibility for their own learning. This ownership becomes

apparent as students are required, in a PBL environment, to actively rather than passively engage in the learning process. Part of being actively engaged is the incorporation of discussion during and after collaborative group work. The sessions might then conclude with projects and/or presentations alongside reflective discussion as students demonstrate their understanding of the concept or problem.

Place Value (PV)

Central to the Number strand of the National Council of Teachers of Mathematics' standards is the development of number sense. Students with number sense naturally decompose numbers, solve problems using the relationships of the base-ten system, estimate a sensible result for a problem, and have a developed ability to make sense of numbers (National Council of Teachers of Mathematics, 2000). Unfortunately, content knowledge of PV systems is a common problem area experienced by university pre-service teachers and their conceptual understanding of PV is underdeveloped (Taplin, 1992).

Context of the Study

This paper focuses on pre-service teachers' conceptual understanding and Mathematical Content Knowledge (MCK) with respect to PV systems. This paper reports the results obtained from an analysis of two exam questions categorising students' MCK of the symmetry of the PV using the SOLO Taxonomy (Biggs & Moore, 1993).

The SOLO taxonomy is a classification system which is used to evaluate the quality of learning outcomes. As an evaluation instrument, it distinguishes between and describes different levels of increasing complexity in a student's understanding of a subject along a developmental continuum of five levels: prestructural, unistructural, multistructural, relational, and the extended abstract level.

The participants of this study were first year Bachelor of Education students enrolled in a mathematics education course at a regional Australian university (N=58). The course is based on the two major curriculum strands of numeration and patterns and algebra. The data collected were from two end-of-semester exam questions, from two consecutive 2009 (n=30) and 2010 (n=28) semesters, answered by students who were in attendance both weeks when place value was investigated. In both years, a two-hour whole group lecture was provided in traditional fashion in a lecture theatre followed by a two-hour tutorial session. The difference between the ways the students were taught is that during the tutorials, the 2010 group was taught using a PBL approach and the 2009 group was taught using a traditional instructor-led approach. Each method was designed to probe students' MCK related to their conceptual understanding of PV systems and their symmetry.

Instructional Process in 2009

Following the two-hour instructor-led lecture, the tutorial instructional process began by providing the students with the appropriate manipulative materials (base-10 and base-5 blocks). The first tutorial question asked the students to look at the base-5 and base-10 blocks and to then convert 245_{base10} to its equivalent number in base-5. Through direct observation it was apparent that students possessed an inadequate content and conceptual understanding of PV to work out the solution unassisted. The vast majority easily built 245 in base-10 with the base-10 blocks (Figure 1). However, most could not build an equivalent number in base-5.

Figure 1: Base-10 blocks

Consequently, the process of the conversion was demonstrated, by the instructor, to the students on the whiteboard in symbolic fashion reminiscent of how it was shown in the power point slides of the lecture (Figure 2).

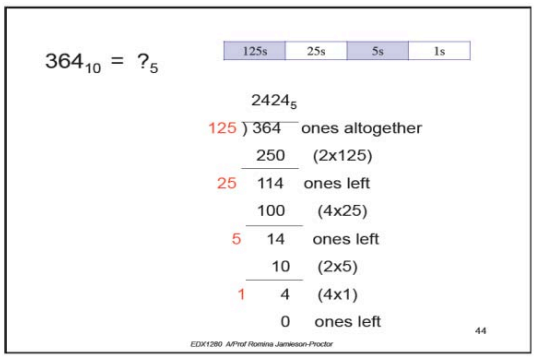

Figure 2: Conversion demonstration

Students were next asked to add $24_{\text{base 5}}$ and $13_{\text{base 5}}$. Once again, most students could not apply the concept of the base-10 system and transfer that knowledge to add two numbers in another base system even with the assistance of concrete materials. In fact, the vast majority of students were not able to set the problem up correctly onto the PV mats. Not surprisingly, students struggled when asked to investigate the differences between the two PV systems. When asked to collaboratively build a pedagogical content knowledge understanding directed around the central question, "How would I help my students understand this" they were unable to respond. As a result, the instructional approach became teacher-centred where the instructor owned the knowledge and provided it to the students through a concrete and symbolic demonstration accompanied by an explanation.

In week two with the 2009 group, students were required to discover and describe the symmetry of different number systems, the role of the decimal point in those systems, the number of different digits needed for each system and the largest digit that could be used in each system.

The first question of the tutorial in week 2 requested students to look at a set of base-10 blocks and draw their representation of what base-2 blocks might look like. Through direct observation it again was apparent that students would need a great deal of guidance and support to be assured they had correctly sketched an accurate representation of base-2.

The next question required students to calculate the base-10 values of binary numbers from a base-2 PV chart (Figure 3). Following that exercise, they were then asked to fill in the missing components of a base-2 PV pattern, describe the pattern, and then extend the pattern on either end by 2 more places.

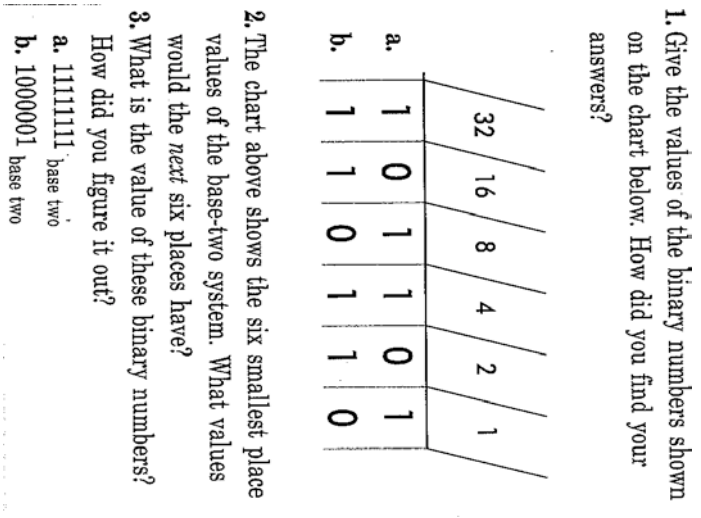

Figure 3: Base-2 PV chart

With minimal guidance by the instructor and having the value of the base-2 houses supplied above question 1.a. in Figure 3, students were, on the whole, successful in calculating the values of each house of the base-2 system. With further explanation and demonstration, students were able to complete the remaining questions of converting the base-2 numbers to values in base-10 (Figure 3).

Following the conversion of base-2 to base-10 questions, students were asked to convert 28_{base10} to a number in base-2. This conversion problem is similar to last week's problem, converting 245_{base10} to its equivalent number in base-5 and met with similar results. By observation it was apparent most students still possessed an inadequate content and conceptual understanding of PV to work out the solution. And once again, the process of the conversion was demonstrated, by the instructor to the students, on the whiteboard in symbolic fashion reminiscent of the lecture's power point slides (Figure 2).

The next question asked students to list as many base-10 PV concepts, or properties, as they could. This question was met primarily with answers from the majority of students that reflected a prestructural or unistructural (Biggs & Moore, 1993) understanding of the concept of PV. In other words, student responses were stated as acquired pieces of disconnected information with little understanding of the core concepts. Only simple and obvious connections were stated if at all. Yet even those were realized comparatively independent of each other and therefore the significance of the connections between their isolated pieces of information were not grasped (Biggs & Collis, 1989).

The final question was a yes or no query which asked, "Could you construct the places for base-3 or base-6 or any base e.g., base-*X*"? Students were not required to demonstrate this, but were alerted to the fact that they might have to answer this on the exam. This was an assumed content knowledge prerequisite for the course. Furthermore, as a methods course, the instructor's objective was to teach mathematics pedagogy not content.

Instructional Process in 2010

For experimental validity purposes, the two hour lecture in 2010 was presented using an archived, recorded version of the 2009 lecture. Also, as in the 2009 cohort, the 2010 group were required to complete all the same tutorial tasks and questions and provided with the same concrete materials. However, this group was instructed using a student-centred, PBL approach. The goal was to guide the students to self-discovery and ownership of the intended learning outcomes through the use of specific PBL activities and key questioning techniques while avoiding providing them with any *teacher-owned* knowledge.

Utilizing the PBL approach, strategically designed tasks preceded several tutorial questions. This strategy required students to complete these specific tasks which tapped into their pre-existing knowledge while also collaborating with their peers. In anticipation of this cohort also possessing an inadequate content and conceptual understanding of PV, a content knowledge intervention task was introduced.

The students were guided through a process where they were to come to their own understanding as to why any number raised to the zero power is 1. The task was to first evaluate 2^7. After the students collaborated, the two common responses received were $2^7 =$ 2 x 2 x 2 x 2 x 2 x 2 x 2 and $2^7 = 128$. Both of these answers were posted on the whiteboard without any elaboration by the teacher.

The students were then asked to evaluate $2^3 \times 2^4$. Again after collaboration, the three agreed upon responses were (2 x 2 x 2) x (2 x 2 x 2 x 2), 8 x 16 and 128. These responses were also posted on the whiteboard without comment by the teacher.

The next task in this self-discovery activity asked students to evaluate $2^{(3+4)}$. As anticipated, responses to this evaluation yielded answers such as (2 x 2 x 2) x (2 x 2 x 2 x 2), 8 x 16 = 128, and as hoped, $2^3 \times 2^4 = 2^7$. These too were posted on the board for analysis and reflection.

The final question in this task required students to explain why, if $2^{(3+4)} = 2^7$, what the value of $2^{(3+0)}$ was. After directing the students to use the previously posted problems and solutions to analyse this problem, the students collaborated and responded that if $2^{(3+4)} = 2^3 \times 2^4$ which is $= 2^7$, then $2^{(3+0)}$ must $= 2^3$. This conclusion then led to the final question, "If $2^3 =$ 8, and $2^3 \times 2^0 = 8$, then what must 2^0 equal"? This activity produced what the researcher likes to call an "ah ha" moment or moment of enlightenment in which the students synthesized for themselves that any number raised to the zero power is one.

Guided through another series of self-directed learning activities, students were challenged to transfer their existing knowledge of the base-10 system to discover answers to base-5, base-2 and base-P type queries which were systematically and strategically provided by the instructor. This PBL activity first required students, in collaborative groups, to reinforce their understanding of the symmetry of the base-10 system by filling in the missing cells of a pre-designed template (Table 1).

Table 1: Base-10 pattern system chart

	10^2	10^1	10^0	10^{-1}		
1000		10			1/100	
10 x 10 x 10		10				1/(10 x 10 x 10)
Thousands	Hundreds	Tens				Thousandths

To enable students to generalize their understanding of the base-10 system, a similar template was completed in collaborative groups on the base-5 system and reflected on through class discussion. Further to this activity was a short session on building the number 245 side-by-side in both base-5 and base-10 (Figure 4).

Figure 4: Building the number 245 side-by-side in Base-5 and Base-10

It was anticipated that by this point in the instructional sequence students would have a higher degree of success in completing the first tutorial question which was to convert 245_{base10} to a number in base-5. With their base-5 blocks, students collaborated to build 245 on their newly completed base-5 PV mats. With guided conversation students in the 2010 group were able to visualize and realize, through their own concrete representations, 245_{base10} as 1440_{base5}. This activity was further followed by a guided symbolic demonstration reminiscent of how it was shown in the power point slides during the lecture (Figure 2) to allow for discussion and reflection.

The next tutorial question asked students to add 24_{base5} and 13_{base5}. Using a PBL approach students were asked to first demonstrate adding 24 and 13 in base-10. This scaffolding strategy reinforced an old skill of setting up a basic base-10 addition problem correctly on a base-10 PV mat. Once accomplished, the students were then asked to build the addition problem in base-5 on the base-5 PV mat using the base-10 example as a guide. Students, through collaboration and peer self-check, successfully set-up and completed the addition problem using their base-5 blocks and mats (Figure 5).

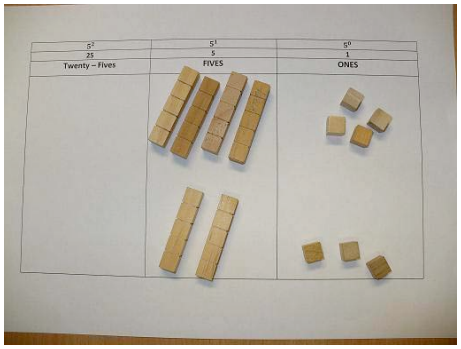

Figure 5: Adding 24_{base5} and 13_{base5}

The next PBL activity was designed to scaffold students' understanding, by building upon the previous week's activities, and then reinforce and extend their understanding of the symmetry of the base-10, base-5, and base-2 system by filling in the missing cells of pre-designed templates (Table2, Table 3, and Table 4).

Table 2: Base-10 pattern system chart [week 2]

	10^2	10^1	10^0			10^{-3}
10 x 10 x 10		10		1/10	1/100	
	100			.1		

Table 3: Base-5 pattern system chart [week 2]

	5^2	5^1	5^0			5^{-3}
5 x 5 x 5		5		1/5	1/(5x5)	
	25			.2		

Table 4: Base-2 pattern system chart

	2^2	2^1	2^0			2^{-3}
2 x 2 x 2		2		1/2	1/(2x2)	
	4			.5		

With the PBL extension activities completed, students were asked to engage in the first tutorial question, "look at the base-10 blocks. Draw your representation of what base-2 blocks might look like." To support this activity, students were provided with a two-

dimensional representation of base-10 and base-5 blocks (Figure 6) and, using a piece of graph paper, directed to generalize and then draw what the base-2 blocks would resemble.

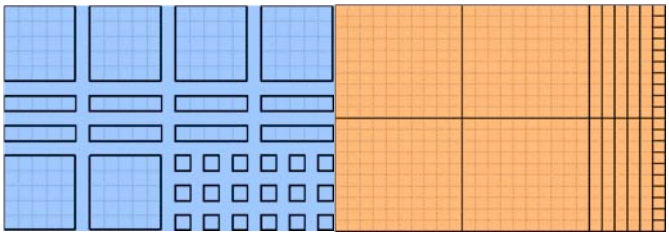

Figure 6: Two dimensional representation of multi-based arithmetic blocks

Completing this task, and the previous task in Table 4, enabled the majority of students to calculate the base-10 equivalency of the binary numbers (Figure 3). Furthermore, they were highly successful in completing the related tutorial tasks of filling in the missing components of the base-2 place value pattern, describing the pattern, and then extending the pattern on either end by 2 more places.

Students were next asked to convert 28_{base10} to a number in base-2. Collaboratively they set up the number 28 onto their base-10 PV mats and then built 28 on their base-2 mats. Consequently, students were able to visualize, realize and then demonstrate the conversion of 28_{base10} to 11100_{base2}. The tutor followed the activity with a guided symbolic demonstration, on the whiteboard, of the conversion process to stimulate discussion and facilitate student reflection of their learning.

The next question asked students to list as many base-10 place value concepts, or properties, as they could. The 2010 tutorial group responded with answers which indicated unistructural, multistructural or relational (Biggs & Moore, 1993) understandings of the concept of PV. In other words, student responses were stated at a level where simple, yet important connections were made as well as responses that indicated an attainment of what could be considered an adequate understanding of the topic (Biggs & Collis, 1989).

The final question asked, "Could you construct the places for base-3 or base-6 or any base e.g., base-*X*"? This question was this time preceded by another activity which required students, in collaborative groups, to reinforce and then extend their understanding of the symmetry of different base systems by filling in the missing cells of a pre-designed template for base-*P* (Table 5).

Table 5: Base-P pattern system chart

	P^2	P^1	P^0			P^{-3}
P x P x P		P				

It was anticipated, based on observation of the students during the tutorial sessions, that the 2010 group given instruction using a PBL approach would demonstrate a higher number of

correct answers on the exam questions versus the 2009 group taught using the teacher-centred approach.

Method of Analysis

This study compared the data obtained from students' responses to two end-of-year semester exam questions, namely:

1. What is your understanding of the term "place value"?
2. Demonstrate your understanding of the symmetry of any place value system by completing all 16 empty cells in Table 6 below.

Table 6: Exam question investigating MCK of place value

Base 10	10^3		10^1	10^0	10^{-1}		10^{-3}
	1000				0.1		0.001
Base 5	5^3		5^1	5^0	5^{-1}	5^{-2}	5^{-3}
		25	5			0.04	
Base a	a^3	a^2			a^{-1}	a^{-2}	a^{-3}
		axa	a		$\frac{1}{a}$	$\frac{1}{axa}$	

The responses to the exam questions were analysed using two different methods. The responses to Question 1 from both cohorts were coded using the SOLO Taxonomy to evaluate student responses. The responses from Question 2 were analyzed using an independent-samples *t*-test to compare the means between the two trial groups by year of the exam. The Statistical Package for the Social Sciences (SPSS) was used to analyse the data.

Results

The students' responses to exam Question 1: *What is your understanding of the term "place value?"* were coded (prestructural=1; unistructural=2; multistructural=3; and relational=4) and compared by year using the Chi-square test to investigate the relationship between the two cohorts. Table 7 provides the percentage of students for each of the first four SOLO levels by year of exam as well as exemplar student responses for each of the four levels.

Table 7: Student response levels for Question 1

Level of Understanding	2009 Group	2010 Group	Indicative Student Responses
Prestructural	9 (30%)	5 (17.9%)	The mother of all mathematics concepts.
Unistructural	19 (63%)	16 (57.1%)	The value of a number is determined by the place it is in.

Multistructural	2 (7%)	7 (25%)	A numbers position determines its value. A numeral can have multiple values.
Relational	0	0	Symmetry runs through the ones house creating a mirror image on either side – tens/tenths. This forms a pattern where each house is ten times larger than the house to the right.
Totals	30 (100%)	28 (100%)	

The Chi-square test indicated a non-significant difference between the two year groups' level of understanding. As can be seen in Table 7 however, more students from the 2010 cohort provided a multistructural response and less student responses were prestructural (meaning they provided a totally incorrect response). In teaching terms, one hopes that an intervention such as PBL will impact positively on student learning outcomes. The researchers believe, based on these results, as well as observations of the students working on the set tutorial tasks each year, that the change in pedagogy to a PBL approach shows promise, even though the result in this study was not statistically significant.

Each of the 16 empty cells in Question 2 (Table 6) was given 1 mark for a correct answer and a 0 for an incorrect response. When the overall scores out of 16 were compared by year group, the results again indicated a non-significant difference between the year groups. Table 8 provides the *mean* and standard deviation for each cohort's overall score out of 16.

Table 8: Overall mean scores for each cohort

	Year	**N**	**Mean**	**Std. Deviation**
Sum of Scores	2009	30	12.40	3.76
	2010	28	13.75	1.71

Finally, Table 9 provides the number and percent of students in each cohort who were able to complete the pattern by naming or calculating the value for each of the three place value systems (Table 6*)*. Students had most difficulty with the fraction places and with naming the ones place. They also had less success when asked to generalise their understanding by naming the bases in a base *a* system.

Table 9: Student responses to exam Question 2

Missing Cells-Table 6	**Correct/Incorrect**	**2009 Group**	**2010 Group**
10^2	Correct/ Incorrect	28 (93%) 2 (7%)	28 (100%) 0
100	Correct/ Incorrect	28 (93%) 2 (7%)	28 (100%) 0
10	Correct/	28 (93%)	28 (100%)

	Incorrect	2 (7%)	0
1	Correct/ Incorrect	24 (80%) 6 (20%)	26 (93%) 2 (7%)
10^{-2}	Correct/ Incorrect	28 (93%) 2 (7%)	28 (100%) 0
0.01	Correct/ Incorrect	26 (87%) 4 (13%)	27 (96%) 1 (4%)
125	Correct/ Incorrect	27 (90%) 3 (10%)	27 (96%) 1 (4%)
5^2	Correct/ Incorrect	28 (93%) 2 (7%)	28 (100%) 0
1	Correct/ Incorrect	24 (80%) 6 (20%)	24 (86%) 4 (14%)
0.2	Correct/ Incorrect	5 (17%) 25 (83%)	6 (21%) 22 (79%)
0.008	Correct/ Incorrect	3 (10%) 27 (90%)	5 (18%) 23 (72%)
a* x *a* x *a	Correct/ Incorrect	25 (83%) 5 (17%)	28 (100%) 0
a^1	Correct/ Incorrect	28 (93%) 2 (7%)	27 (96%) 1 (4%)
a^0	Correct/ Incorrect	28 (93%) 2 (7%)	27 (96%) 1 (4%)
1	Correct/ Incorrect	20 (67%) 10 (33%)	21 (75%) 7 (25%)
1/(*a* x *a* x *a*)	Correct/ Incorrect	22 (73%) 8 (27%)	27 (96%) 1 (4%)

Conclusion

It has been long recognised that "...the real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher's understanding of mathematics" (Ma, 1999). Unfortunately, many soon-to-be teachers, as this study further demonstrates, have limited understanding of mathematical concepts such as place value. In this study both cohorts' MCK was weakest when working with decimal fractions and when generalising their understanding of place value to any base. No students in either cohort, irrespective of teaching method, appear to possess a relational understanding of place value.

However, the PBL pedagogy may be responsible for the increase in the number of students in the 2010 cohort who appear to possess a multistructural understanding of the place value concept. Unfortunately, due to the relatively small sample size of each year group, a causal relationship cannot be drawn from this study. What can be stated clearly is that pre-service teachers have a less than desirable understanding of place value and ability to operationalise this understanding by naming the places in different bases. This lack of MCK with respect to place value needs to be addressed in order that their misconceptions are not transmitted to their students after they graduate. Hence, it may be suggested from this study and from other similar studies that if teacher knowledge of subject matter, student learning, and teaching methods are all important elements of teacher effectiveness (National Commission on Teaching and America's Future, 1996); and, effective teaching begins with effective teacher

preparation, then university teacher preparation programs should focus their efforts on ensuring that pre-service education graduates have strong MCK and are equipped to use research-based instructional strategies such as the problem-based learning approach (Miller, 2003).

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